International Journal of Advances in Engineering and Management (IJAEM)
Volume 2, Issue 4, pp: 771-796 www.ijaem.net ISSN: 2395-5252

# * Free Convection Heat and Mass Transfer Flow past through a porous medium between two long vertical wavy walls in a viscous incompressible fluid.

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Date of Submission: 02-08-2020 Date of Acceptance: 20-08-2020

**ABSTRACT:** The study of two dimensional free convective heat and mass transfer flow of a viscous incompressible and electrically conducting fluid in a porous medium confined between two long vertical wavy walls has been studied under the assumption that the wave length of the wavy walls are large. Consider that the wavy walls have different amplitude. A uniform magnetic field is assumed to be applied perpendicular to the walls in the absence of waviness.
The velocity field, temperature, concentration, skin friction, Nusselt number and Sherwood number for different values of the parameters involved in the problem are obtained. The dimensionless governing equations are solved analytically by using regular perturbation technique subject to the relevant boundary conditions and are presented graphically
**Keywords**: Nusselt number, skin friction, magnetic field, heat transfer, mass transfer, porous medium, velocity field.

## I. INTRODUCTION

The principle of MHD is utilized in stabilizing a flow against the transition from laminar ton turbulent flow and in reduction of turbulent drag and suppression of flow separation. Viscous flow (fluid) over a wavy walls has attracted the attention of relatively few researchers although the analysis of such flows finds applications in different areas such as transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers flows, which arise in fluids due to the interaction of the force of gravity and density difference caused by the simultaneous diffusion of the thermal energy and chemical species have many application in Geophysics and Engineering. The study through a porous medium has got importance because of its occurrence in movement of water and oil inside the Earth, flow of river through porous banks, chemical engineering for filtration and purification process, petroleum technology, to study the movement of natural gas and in the fields of agriculture engineering to study the underground water resources. Lesson and Gangwati (1976) have analyzed the effect of small amplitude wall waviness upon the stability of the laminar boundary layer. Vajravelu and Sastri (1978) have made an analysis of the free convective heat transfer in a viscous incompressible fluid between a long vertical wavy walls and a parallel flat wall. Vajravelu and Sastri (1980) have extended their work for the case when the channel walls are wavy. Patidar and Purohit have studied the flow of viscous incompressible fluid in porous medium confined between two long vertically wavy walls when the amplitude of the waviness of both the walls is different. Rao and Sastri (1982) have reinvestigated the work of Vajravelu and Sastri when the viscous heating effects are considered and when the fluid properties are constant and which are variables. It has been solved the equations by Galerkin’s method. Rao (1982) analyzed the problem studied by Rao and Sastri to the case when the channels are different wavy numbers. Taneja and Jain (2004) studied the problem of MHD flow with slip effects and temperature dependent heat source in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. Ahmed et al. (2005) have studied the problem of Patidar and Purohit in the presence of transverse magnetic field. Alam and Rahman (2005) have studied the Dufour and Soret effects on MHD free convective heat and mass transfer flow past a vertical porous plate embedded in a porous medium. Kafoussias and Williams (1995) studied the effects of thermal diffusion and diffusion-thermo on steady mixed free-forced convective and mass transfer over a vertical flat plate, when the viscosity of the fluid is varies with temperature. Chamkha and Ben-Nachi (2008) have studied the MHD mixed convection flow under radiation interaction along a vertical permeable surface immersed in a porous medium in the presence of Soret and Dufour effects.

The mass transfer caused by the temperature gradient is called Soret effect and it has utilized for isotope separation and in a mixture between gases and with very light molecular weight ($H_2$, He) and of medium molecular weight ($N_2$, air), while the heat transfer caused by the concentration gradient is called the Dufour effect. Recently, Kumar (2011) studied the problem of heat transfer with radiation and temperature dependent heat source in MHD free convection flow confined between two vertical wavy walls. Again, Ahmed (2010) investigated the MHD free convection with Soret and Dufour effects in a three dimensional flow past an infinite vertical porous plate. Chamkha et al. (2011) have studied the problem of unsteady double diffusive natural convective MHD flows along a vertical cylinder in presence of chemical reaction, thermal radiation and Soret and Dufour effects. Recently, Pandit et al. (2015) investigated the effect of chemical reaction and thermal radiation on unsteady MHD free convection heat and mass transfer of an electrically conducting viscous incompressible and heat absorbing fluid. Elbashbeshy et al. (2010) have studied the problem of heat transfer over an unsteady porous stretching surface embedded in a porous medium with variable heat flux in the presence of heat source or sink. Rashad (2014) studied the effect of radiation and variable viscosity on unsteady MHD flow of a rotating fluid from stretching surface in porous medium.

The objective of the present work is to study free convective heat and mass transfer flow of a viscous incompressible fluid in a porous medium between two long vertical wavy wall.

## 1. Mathematical Formulation:

Let us consider the two dimensional free convective MHD flow of a viscous incompressible electrically conducting fluid between two long vertical, wavy non-electrically conducting walls in porous medium. Here x-axis is taken vertically upward and parallel to the flat wall and y-axis is perpendicular to it. The wavy walls are represented by $\bar{y}=\bar{\varepsilon}\cos\left(\lambda\bar{x}\right)$ and $\bar{y}=d\left(1+h\bar{\varepsilon}\cos\left(\lambda\bar{x}\right)\right)$, where d is the distance between the two walls. The governing equations of the problem are

Equation of Continuity:

$$\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{y}}=0 \tag{1}$$

Conservation of Momentum:

$$\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}=-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{x}}+\upsilon\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right)+g\beta\left(\bar{T}-\bar{T}_S\right)+g\bar{\beta}\left(\bar{C}-\bar{C}_S\right)-\frac{\upsilon\bar{u}}{k}-\frac{\sigma B_0^2\bar{u}}{\rho} \tag{2}$$

$$\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}=-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{y}}+\upsilon\left(\frac{\partial^2\bar{v}}{\partial\bar{x}^2}+\frac{\partial^2\bar{v}}{\partial\bar{y}^2}\right)-\frac{\upsilon\bar{v}}{k} \tag{3}$$

Conservation of Energy:

$$\bar{u}\frac{\partial\bar{T}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{T}}{\partial\bar{y}}=a\left(\frac{\partial^2\bar{T}}{\partial\bar{x}^2}+\frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right) \tag{4}$$

Conservation of Species Concentration:

$$\bar{u}\frac{\partial\bar{C}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{C}}{\partial\bar{y}}=D\left(\frac{\partial^2\bar{C}}{\partial\bar{x}^2}+\frac{\partial^2\bar{C}}{\partial\bar{y}^2}\right) \tag{5}$$

The boundary conditions relevant to the problem are taken as:

$$\bar{u}=0,\bar{v}=0,\bar{T}=\bar{T}_W,\bar{C}=\bar{C}_W \text{ at } \bar{y}=\bar{\varepsilon}\cos\left(\lambda\bar{x}\right) \tag{6a}$$

$$\bar{u}=0,\bar{v}=0,\bar{T}=\bar{T}_1,\bar{C}=\bar{C}_1 \text{ at } \bar{y}=d\left(1+h\bar{\varepsilon}\cos\left(\lambda\bar{x}\right)\right) \tag{6b}$$

where h is the amplitude parameter for the second wavy walls.

In order to reduce the governing equations (1)-(5) into non-dimensional form, the following dimensionless variables and parameters are introduced.

$$x=\frac{\bar{x}}{d},y=\frac{\bar{y}}{d},u=\frac{\bar{u}d}{\upsilon},v=\frac{\bar{v}d}{\upsilon},p=\frac{\bar{p}d^2}{\rho\upsilon^2},T=\frac{\bar{T}-\bar{T}_S}{\bar{T}_w-\bar{T}_S},C=\frac{\bar{C}-\bar{C}_S}{\bar{C}_w-\bar{C}_S},\alpha=\frac{d}{\sqrt{k}},M=\frac{\sigma B_0^2 d^2}{\rho\upsilon},Gr=\frac{g\beta d^3\left(\bar{T}_w-\bar{T}_S\right)}{\upsilon^2},$$

$$Gm=\frac{g\bar{\beta}d^3\left(\bar{C}_w-\bar{C}_S\right)}{\upsilon^2},\Pr=\frac{\upsilon}{a},\mathrm{P}m=\frac{\upsilon}{D},\varepsilon=\frac{\bar{\varepsilon}}{d},\lambda=\bar{\lambda}d,m=\frac{\bar{T}_1-\bar{T}_S}{\bar{T}_w-\bar{T}_S},\alpha_1=hd,n=\frac{\bar{C}_1-\bar{C}_S}{\bar{C}_w-\bar{C}_S}.$$

Equations (1)-(5) reduces to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial x}+\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+GrT+GmC-\alpha^2 u-Mu \tag{8}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\alpha^2 v \tag{9}$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\Pr\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right] \tag{10}$$

$$\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}=\mathrm{P}m\left[u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right] \tag{11}$$

The corresponding boundary conditions in non-dimensional form become:

$$u=0,v=0,T=1,C=1 \text{ at } y=\varepsilon\cos\left(\lambda x\right) \tag{12a}$$

$$u=0,v=0,T=m,C=n \text{ at } y=1+\alpha_1\varepsilon\cos\left(\lambda x\right) \tag{12b}$$

## 2. Method of Solutions:

In order to solve equations (7)-(11), let us assume that the solution consists of two parts i.e., mean part and a perturbed part as given below:

$$\left.\begin{aligned} u(x,y)&=u_0(y)+\varepsilon u_1(x,y)\\ p(x,y)&=p_0(y)+\varepsilon p_1(x,y)\\ C(x,y)&=C_0(y)+\varepsilon C_1(x,y)\\ v(x,y)&=\varepsilon v_1(x,y)\\ T(x,y)&=T_0(y)+\varepsilon T_1(x,y)\end{aligned}\right\} \tag{13}$$

where $u_0,p_0,C_0,T_0$ and $u_1,p_1,C_1,T_1$ are the mean and perturbed part of velocity, pressure, concentration and temperature respectively.

Substituting (13) in (7)-(11) and equating co-efficient of $\varepsilon^0,\varepsilon^1$ and neglecting higher order terms we have the following system of equations:

Zeroth-Order equations:

$$\frac{\partial u_0}{\partial x}=0 \tag{14}$$

$$\frac{\partial^2 u_0}{\partial y^2}-\left(\alpha^2+M\right)u_0=-GrT_0-GmC_0 \tag{15}$$

$$\frac{\partial p_0}{\partial y}=0 \quad (16)$$

$$\frac{\partial^2 T_0}{\partial y^2}=0 \quad (17)$$

$$\frac{\partial^2 C_0}{\partial y^2}=0 \quad (18)$$

First order Equations:

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial x}=0 \quad (19)$$

$$\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial y^2}-u_0\frac{\partial u_1}{\partial x}-\lambda_1^2 u_1=\frac{\partial p_1}{\partial x}+v_1\frac{\partial u_0}{\partial y}-GrT_1-GmC_1 \quad (20)$$

$$\frac{\partial^2 v_1}{\partial x^2}+\frac{\partial^2 v_1}{\partial y^2}-u_0\frac{\partial v_1}{\partial x}-\alpha^2 v_1=\frac{\partial p_1}{\partial y} \quad (21)$$

$$\frac{\partial^2 T_1}{\partial x^2}+\frac{\partial^2 T_1}{\partial y^2}=\Pr\left(u_0\frac{\partial T_1}{\partial x}+v_1\frac{\partial T_0}{\partial y}\right) \quad (22)$$

$$\frac{\partial^2 C_1}{\partial x^2}+\frac{\partial^2 C_1}{\partial y^2}=\mathrm{P}m\left(u_0\frac{\partial C_1}{\partial x}+v_1\frac{\partial C_0}{\partial y}\right) \quad (23)$$

$$\lambda_1^2=\alpha^2+M$$

Zeroth order Boundary conditions are

$$u_0=0, T_0=1, C_0=1 \text{ at } y=0 \quad (24a)$$

$$u_0=0, T_0=m, C_0=n \text{ at } y=1 \quad (24b)$$

The equations (14) to (18) subject to the boundary conditions (24a) and (24b) have been solved and the solutions of the zeroth order equations (14) to (18) are given in the following form:

$$T_0(y)=1+(m-1)y \quad (25)$$

$$C_0(y)=1+(n-1)y \quad (26)$$

$$u_0(y)=V_{11}e^{\lambda_1 y}+V_{22}e^{-\lambda_1 y}+\frac{Gr}{\lambda_1^2}\left[(m-1)y+1\right]+\frac{Gm}{\lambda_1^2}\left[(n-1)y+1\right] \quad (27)$$

First order Boundary conditions are

$$u_1=-\frac{du_0}{dy}\cos(\lambda x), v_1=0, T_1=-\frac{dT_0}{dy}\cos(\lambda x), C_1=-\frac{dC_0}{dy}\cos(\lambda x) \text{ at } y=0 \quad (28a)$$

$$u_1=-\alpha_1\frac{du_0}{dy}\cos(\lambda x), v_1=0, T_1=-\alpha_1\frac{dT_0}{dy}\cos(\lambda x), C_1=-\alpha_1\frac{dC_0}{dy}\cos(\lambda x) \text{ at } y=1 \quad (28b)$$

We now introduce the stream function $\psi(x,y)$ as

$$u_1=-\frac{\partial \psi_1}{\partial y}, v_1=\frac{\partial \psi_1}{\partial x} \quad (29)$$

The equation of continuity is identically satisfied. On elimination of pressure term from equations (19) and (20), we get

$$\psi_{xxxx}-u_0\psi_{xxx}-\alpha^2\psi_{xx}-u_0\psi_{xyy}-\lambda_1^2\psi_{yy}+\psi_{yyyy}+\psi_x u_0^4=GrT_1(y)+GmC_1(y) \quad (30)$$

Now to solve (22), (23) and (30) we consider

$\psi = e^{i\lambda x}\overline{\psi}(y), T_1 = e^{i\lambda x}\theta(y), C_1 = e^{i\lambda x}\phi(y)$, the equations (22), (23) and (30) reduce to

$$\theta'' - \left(\lambda^2 + i\Pr u_0\lambda\right)\theta = i\Pr\lambda T_0'\overline{\psi} \tag{31}$$

$$\phi'' - \left(\lambda^2 + i\Pr u_0\lambda\right)\phi = i\Pr\lambda C_0'\overline{\psi} \tag{32}$$

$$\overline{\psi}^{iv} - \left(iu_0\lambda + \alpha^2 + \mu\right)\overline{\psi}'' + \left(iu_0\lambda^3 + \lambda^4 + \alpha^2\lambda^2 + i\lambda u_0''\right)\overline{\psi} = Gr\theta' + Gm\phi' \tag{33}$$

The corresponding boundary conditions are

$$\overline{\psi}' = \alpha\left(c_2 - c_1\right) + \frac{Gr}{\alpha^2}(m-1) + \frac{Gm}{\alpha^2}(n-1), \overline{\psi} = 0, \theta = 1-m, \phi = 1-n \text{ at } y = 0 \tag{34a}$$

$$\overline{\psi}' = \alpha_1\left[c_2\alpha e^{\alpha} - c_1\alpha e^{-\alpha} + \frac{Gr}{\alpha^2}(m-1) + \frac{Gm}{\alpha^2}(n-1)\right], \overline{\psi} = 0, \theta = \alpha_1(1-m), \phi = \alpha_1(1-n) \text{ at } y = 1 \tag{34b}$$

where λ is small

Substituting $\overline{\psi} = \sum_i \lambda^i \overline{\psi}_i, \theta = \sum_i \lambda^i \theta_i, \phi = \sum_i \lambda^i \phi_i$ in equations (31)-(33) and comparing the coefficients of like powers of λ we get

$$\theta_0'' = 0 \tag{35}$$

$$\theta_1'' = i\Pr\left(u_0\theta_0 + T_0'\overline{\psi}_0\right) \tag{36}$$

$$\phi_0'' = 0 \tag{37}$$

$$\phi_1'' = i\Pr\left(u_0\phi_0 + C_0'\overline{\psi}_0\right) \tag{38}$$

$$\overline{\psi}_0^{iv} - \left(\alpha^2 + M\right)\overline{\psi}_0'' = Gr\theta_0' + Gm\phi_0' \tag{39}$$

$$\overline{\psi}_0^{iv} - iu_0\overline{\psi}_0'' - \left(\alpha^2 + M\right)\overline{\psi}_1'' + iu_0''\overline{\psi}_0 = Gr\theta_1' + Gm\phi_1' \tag{40}$$

The corresponding boundary conditions are

$$\overline{\psi}' = \alpha\left(c_2 - c_1\right) + \frac{Gr}{\alpha^2}(m-1) + \frac{Gm}{\alpha^2}(n-1), \overline{\psi}_0 = 0, \theta_0 = 1-m, \phi_0 = 1-n \text{ at } y = 0 \tag{41a}$$

$$\overline{\psi}' = \alpha_1\left[c_2\alpha e^{\alpha} - c_1\alpha e^{-\alpha} + \frac{Gr}{\alpha^2}(m-1) + \frac{Gm}{\alpha^2}(n-1)\right], \overline{\psi}_0 = 0, \theta_0 = \alpha_1(1-m), \phi_0 = \alpha_1(1-n) \text{ at } y = 1 \tag{41b}$$

The solution of the equations (35) to (40) under the boundary conditions (41a) and (41b) are given as

$$\theta_0(y) = (1-m)\left[1 + (\alpha_1 - 1)y\right] \tag{42}$$

$$\phi_0(y) = (1-n)\left[1 + (\alpha_1 - 1)y\right] \tag{43}$$

$$\overline{\psi}_0(y) = Z_1 + Z_2 y + Z_3 e^{\lambda_1 y} + Z_4 e^{-\lambda_1 y} + \frac{V_1}{2\lambda_1^2}y^2 + \frac{V_2}{2\lambda_1^2}y^2 \tag{44}$$

$$\theta_1(y) = Z_{14} + Z_{15}y + \frac{i\Pr(1-m)Z_{16}y^2}{2} + \frac{iPm(1-m)Z_{17}y^3}{6} + \frac{i\Pr(1-m)Z_{18}y^4}{12} + \frac{iPm(1-m)Z_{19}}{\lambda_1^2}e^{\lambda_1 y} -$$

$$\frac{2i\Pr(1-m)Z_{20}}{\lambda_1^3}e^{\lambda_1 y} + \frac{iPm(1-m)Z_{21}}{\lambda_1^2}e^{-\lambda_1 y} + \frac{i\Pr(1-m)Z_{22}}{\lambda_1^3}e^{-\lambda_1 y} + \frac{iPm(1-m)Z_{21}}{\lambda_1^2}ye^{\lambda_1 y} + \frac{iPm(1-m)Z_{22}}{\lambda_1^2}ye^{-\lambda_1 y}$$

(45)

$$\phi_1(y)=Z_5+Z_6y+\frac{i\Pr(1-n)Z_7y^2}{2}+\frac{iPm(1-n)Z_8y^3}{6}+\frac{i\Pr(1-n)Z_9y^4}{12}+\frac{iPm(1-n)Z_{10}}{\lambda_1^2}e^{\lambda_1 y}-\frac{2iPm(1-n)Z_{11}}{\lambda_1^3}e^{\lambda_1 y}+\frac{i\Pr(1-n)Z_{12}}{\lambda_1^2}e^{-\lambda_1 y}+\frac{2i\Pr(1-n)Z_{13}}{\lambda_1^3}e^{-\lambda_1 y}+\frac{i\Pr(1-n)Z_{11}}{\lambda_1^2}ye^{\lambda_1 y}+\frac{iPm(1-n)Z_{13}}{\lambda_1^2}ye^{-\lambda_1 y} \quad (46)$$

$$\bar{\psi}_1(y)=V_3+V_4y-V_5y^2-V_6y^3-V_7y^4-V_8y^5+(V_9+V_{10})e^{\lambda_1 y}+(V_{11}+V_{12})e^{-\lambda_1 y}+V_{13}ye^{\lambda_1 y}-V_{14}ye^{-\lambda_1 y}+V_{15}y^2e^{-\lambda_1 y}-V_{16}y^2e^{-\lambda_1 y} \quad (47)$$

## 3. Skin-Friction, The Rate Of Heat Transfer And The Rate Of Mass Transfer:

The non-dimensional shear stress $\tau$ at the plate y=0 in the direction of free stream is given by:

$$\bar{\tau}_{xy}=\frac{d^2\tau_{xy}}{\rho v^2}$$

$$\bar{\tau}_{xy}=\mu\left(\frac{\partial\bar{u}}{\partial\bar{y}}+\frac{\partial\bar{v}}{\partial\bar{x}}\right)$$

$$=u_0'(0)+\varepsilon e^{i\lambda x}u_1'(0)+i\varepsilon\lambda e^{i\lambda x}v_1'(0)$$

The skin friction at the wavy wall $y=\varepsilon\cos(\lambda x)$ is given by,

$$\tau_{w_1}=\tau_0^0+\varepsilon\,\mathrm{Re}\left[e^{i\lambda x}u_0''(0)+e^{i\lambda x}u_1''(0)\right]$$

where $\tau_0^0=u_0'(0)$

$$=\lambda_1V_{11}-\lambda_1V_{22}+\frac{Gr}{\lambda_1^2}(m-1)+\frac{Gm}{\lambda_1^2}(n-1)$$

The skin friction at the wavy wall $y=1+\alpha_1\varepsilon\cos(\lambda x)$ is given by,

$$\tau_{w_2}=\tau_1^0+\mathrm{Re}\left[\varepsilon e^{i\lambda x}u_1'(1)\right]$$

where $\tau_1^0=u_0'(1)$

$$=\lambda_1V_{11}e^{\lambda_1}-\lambda_1V_{22}e^{-\lambda_1}+\frac{Gr}{\lambda_1^2}(m-1)+\frac{Gm}{\lambda_1^2}(n-1)$$

The dimensionless heat flux in terms of Nusselt number Nu is given by

$$Nu=\frac{\partial\theta}{\partial y}=\theta_0'(y)+\varepsilon\theta_1'(y)$$

The Nusselt number Nu at the wavy wall $y=\varepsilon\cos(\lambda x)$ is given by,

$$Nu_{w_1}=Nu_0^0+\varepsilon\,\mathrm{Re}\left[\theta_0''(0)+\theta'(0)\right]$$

where $Nu_0^0=\theta_0'(0)=(1-m)(\alpha_1-1)$

The Nusselt number Nu at the wavy wall $y=1+\alpha_1\varepsilon\cos(\lambda x)$ is given by,

$$Nu_{w_2}=Nu_1^0+\varepsilon\,\mathrm{Re}\,\theta'(1)$$

where $Nu_1^0=\theta_0'(1)=(1-m)(\alpha_1-1)$

The dimensionless mass flux in terms of Sherwood number Sh is given by

$$Sh = \frac{\partial \phi}{\partial y} = \phi_0'(y) + \varepsilon \phi_1'(y)$$

The Sherwood number Sh at the wavy wall $y = \varepsilon \cos(\lambda x)$ is given by,

$$Sh_{w_1} = Sh_0^0 + \varepsilon \operatorname{Re}\left[\phi_0''(0) + \phi'(0)\right]$$

where $Sh_0^0 = \phi_0'(0) = (1-n)(\alpha_1 - 1)$

The Sherwood number Sh at the wavy wall $y = 1 + \alpha_1 \varepsilon \cos(\lambda x)$ is given by,

$$Sh_{w_2} = Sh_1^0 + \varepsilon \operatorname{Re} \phi'(1)$$

where $Sh_1^0 = \phi_0'(1) = (1-n)(\alpha_1 - 1)$

## II. RESULTS AND DISCUSSIONS:

In order to study the physical insight into the problem, we have carried out numerical calculations for non-dimensional velocity field (u), temperature field (θ), species concentration (ϕ), skin friction (τ), Nusselt number (Nu), Sherwood number (Sh) at both the wavy walls. It is assigning some specific values to the parameters entering into the problem and effects of these values on the above field are discussed. We have restricted our investigation to Prandtl number (Pr) is taken to be 0.71 which corresponds to air and the other parameters. In this problem the values of the frequency parameter λ, amplitude parameter ε and λx are kept fixed at 0.01, 0.001 and $\pi/2$ respectively.

The behaviour of the fluid velocity u under Hartmann number (M), solutal Grashof number (Gm), thermal Grashof number (Gr), heat source parameter (α) wall temperature ratio (m) and wall concentration ratio (n) is shown in Figs. 3-7 respectively. It is observed that solutal Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. The fluid velocity increases due to increase in the species buoyancy force. The thermal Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. It is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. It is seen that increasing values of α, there is increase in the thickness of velocity boundary layer. Here, the positive values of Gr correspond to the cooling of the plate.

In Fig.8, the variation of velocity for different values Hartmann number M is shown it is seen that fluid velocity decreases for increasing value of M.

Figs. 9-11, shows the behaviour of skin friction (τ) against Hartmann number (M) under the influence of heat source parameter (α), Grashof number for heat transfer (Gr), and Grashof number for mass transfer (Gm). The magnitude of the viscous drag decreases for increasing values of Grashof number for heat transfer (Gr), Grashof number for mass transfer (Gm) and heat source parameter (α).

In Figs.12-14, the different values of heat source parameter (α), Grashof number for mass transfer (Gm), and Grashof number for heat transfer (Gr) are plotted. It is seen that increasing values of α, Gr and Gm, there is an increase in the thickness of temperature boundary layer.

In Figs. 15-17, the different values of heat source parameter (α), Grashof number for mass transfer (Gm), and Grashof number for heat transfer (Gr) are plotted. It is seen that increasing values of α, Gr and Gm, there is a decrease in the thickness of concentration boundary layer.

The behaviour of Nusselt number (Nu) against wall temperature ratio (m) and amplitude parameter $(\alpha_1)$ can be observed in the Figs. 18 and 19.

Figs. 20 and 21, display the effect of wall concentration ratio (n) and amplitude parameter $(\alpha_1)$ on the rate of mass transfer. The rate of mass transfer increases for second wavy wall but decreases due to mass diffusion. In case of first wavy wall, the rate of mass transfer shows a reverse trend

International Journal of Advances in Engineering and Management (IJAEM)
Volume 2, Issue 4, pp: 771-796 www.ijaem.net ISSN: 2395-5252

**FIGURES**

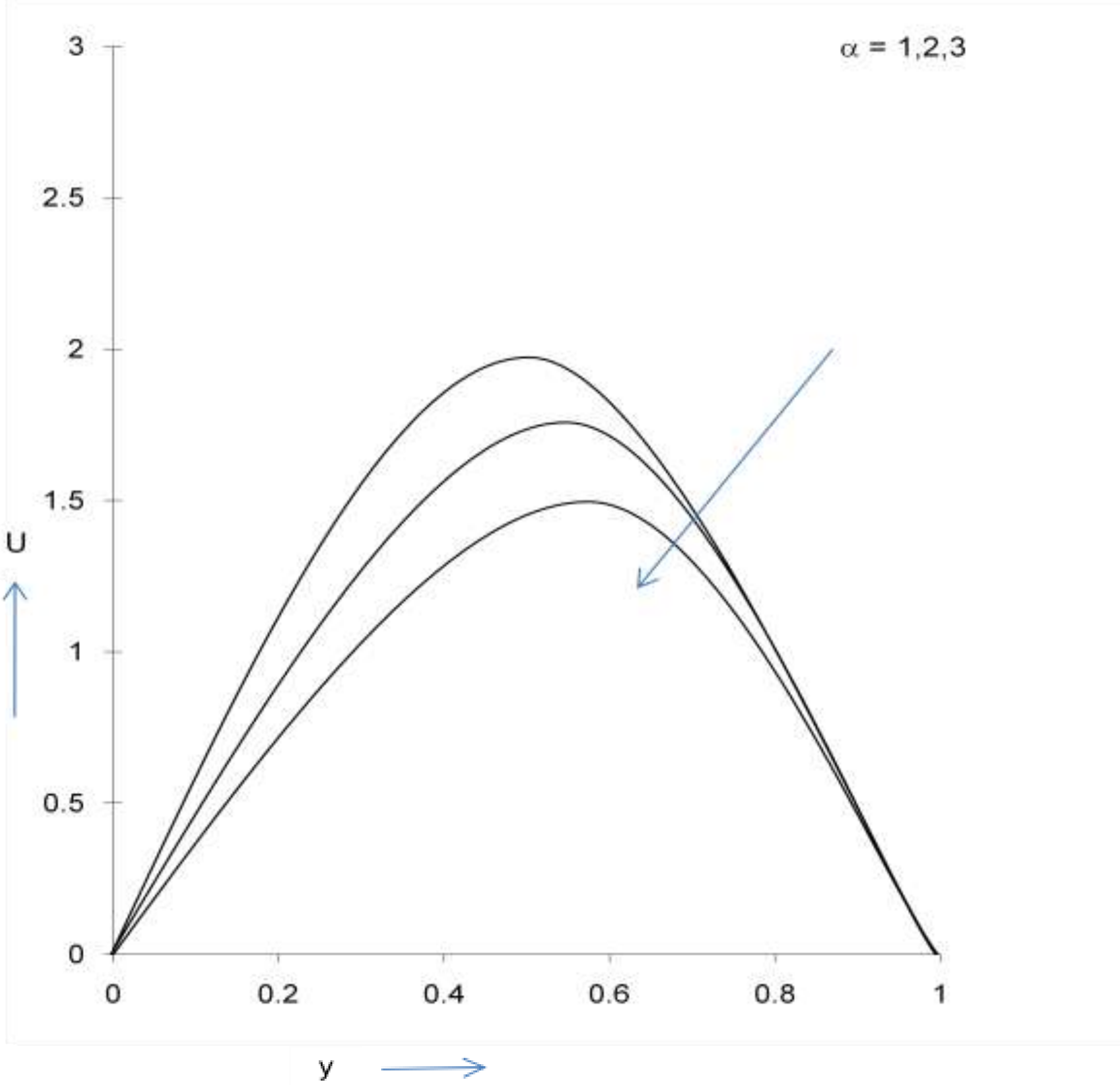


**Fig 1:** Velocity profile for different values of α for M=2, Gr=2, Gm=3, m=.2, n=.4

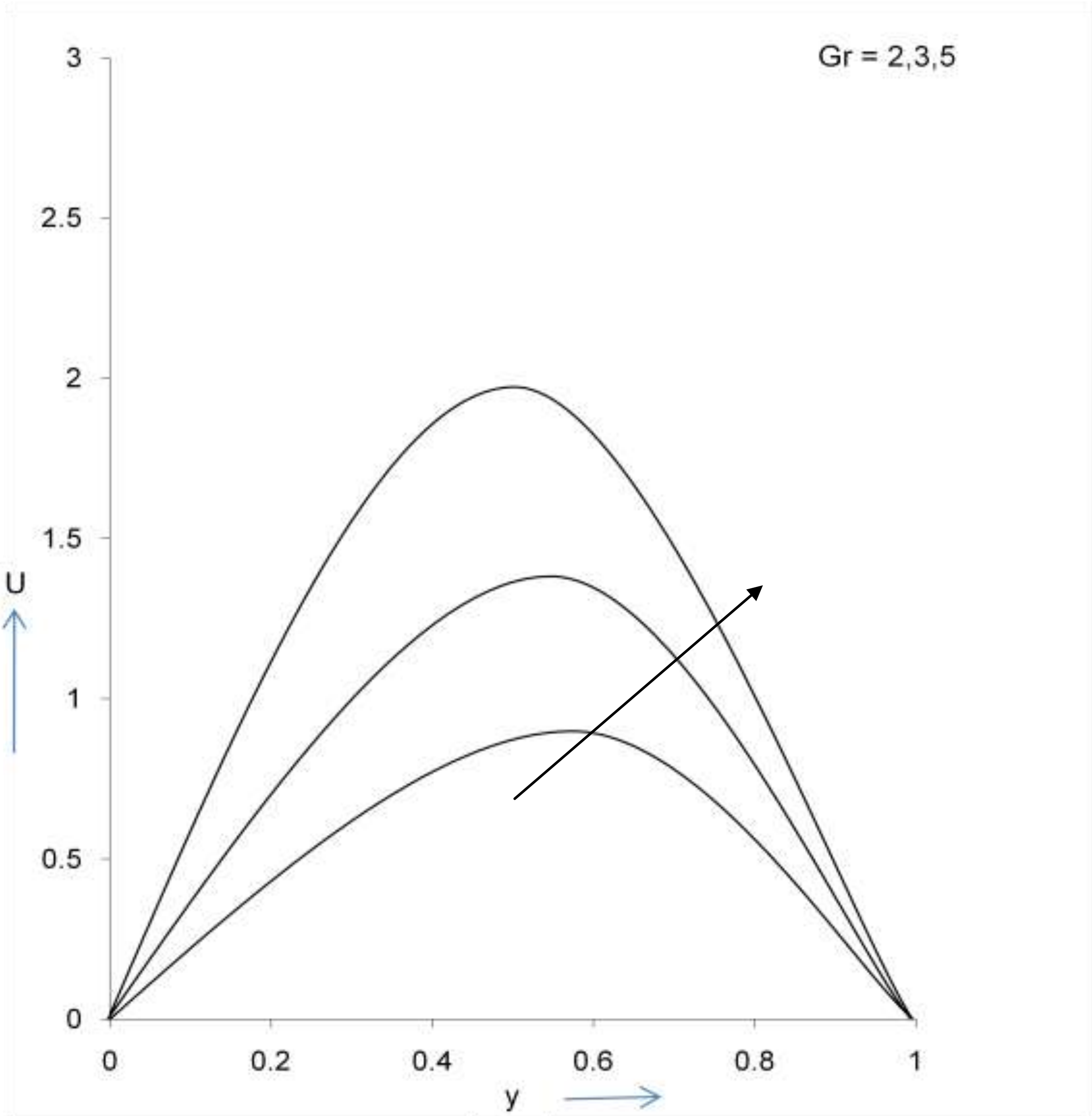


**Fig 2:** Velocity profile for different values of Gr for M=2, Gm=3, m=.2, n=.4, □ □ □

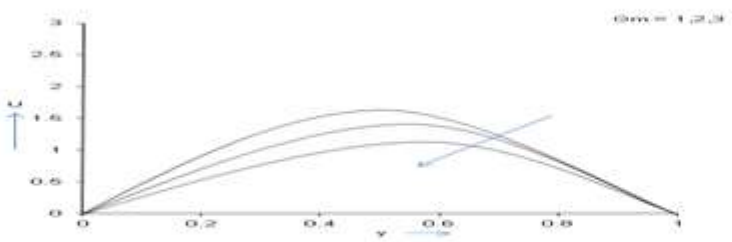

**Fig 3:** Velocity profile for different values of Gm for M=2, Gr=2, m=.2, n=.4, □ □ □

International Journal of Advances in Engineering and Management (IJAEM)
Volume 2, Issue 4, pp: 771-796 www.ijaem.net ISSN: 2395-5252

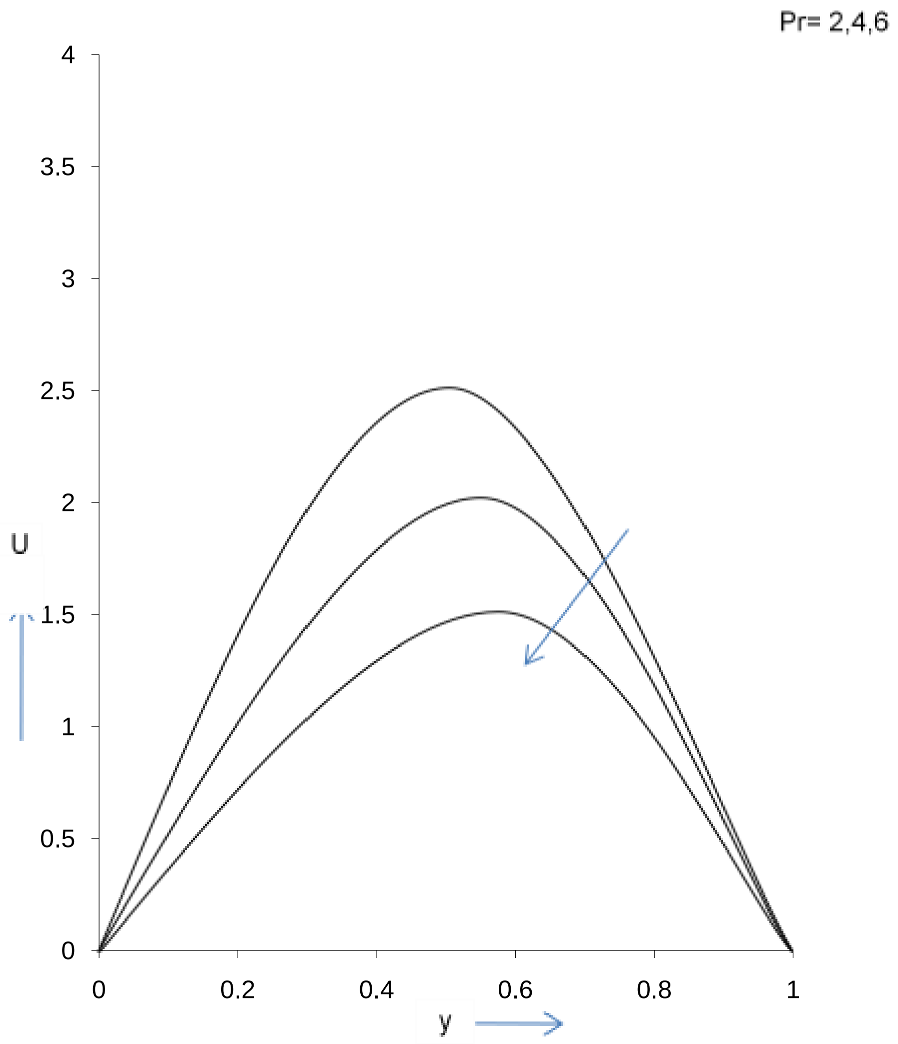


**Fig 4 : Velocity profile for different values of Pr for  =1, Gr=2, Gm=3, m=.2, n=.4**

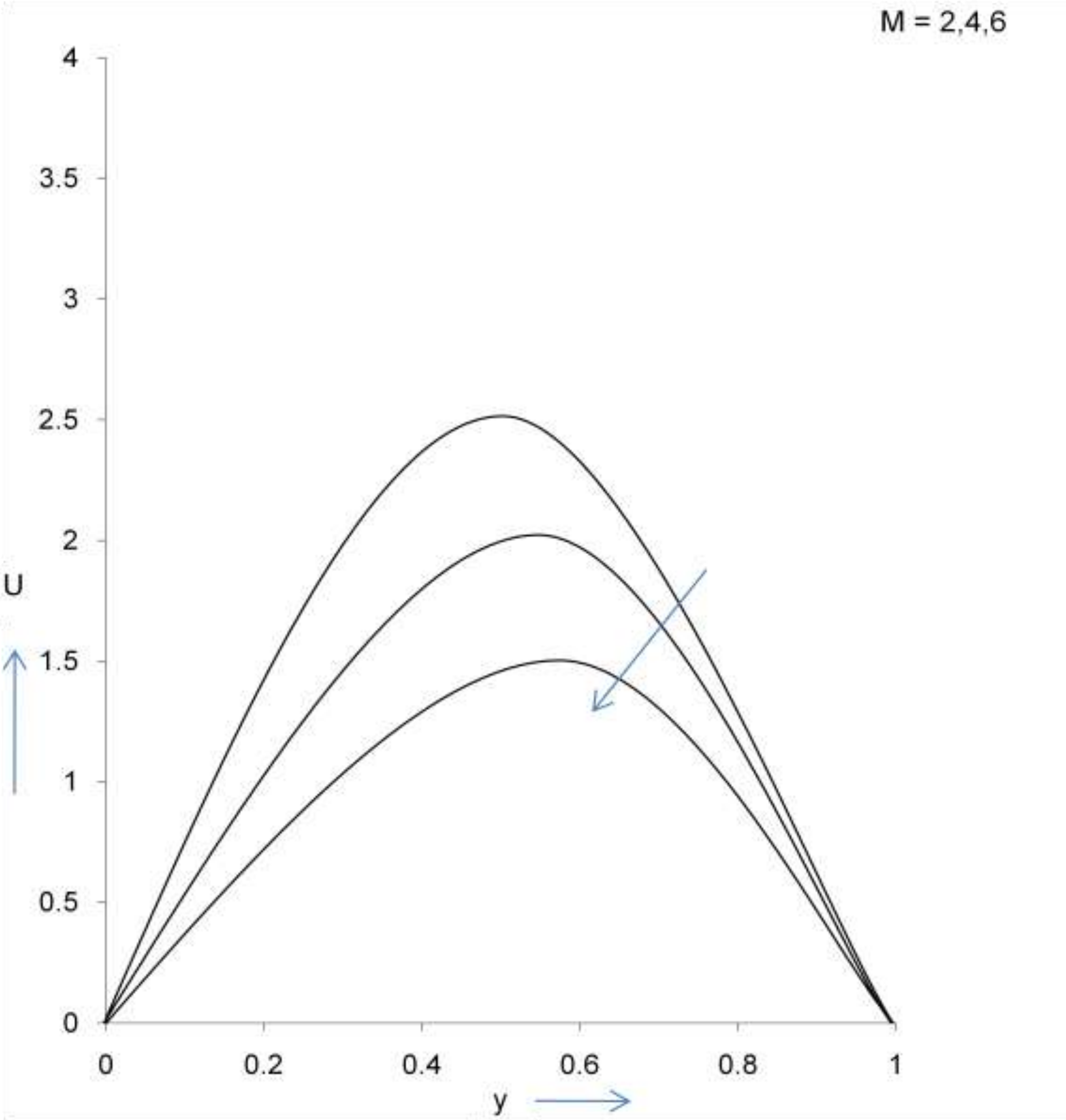


**Fig 5:** Velocity profile for different values of M for =1, Gr=2, Gm=3, m=.2, n=.4

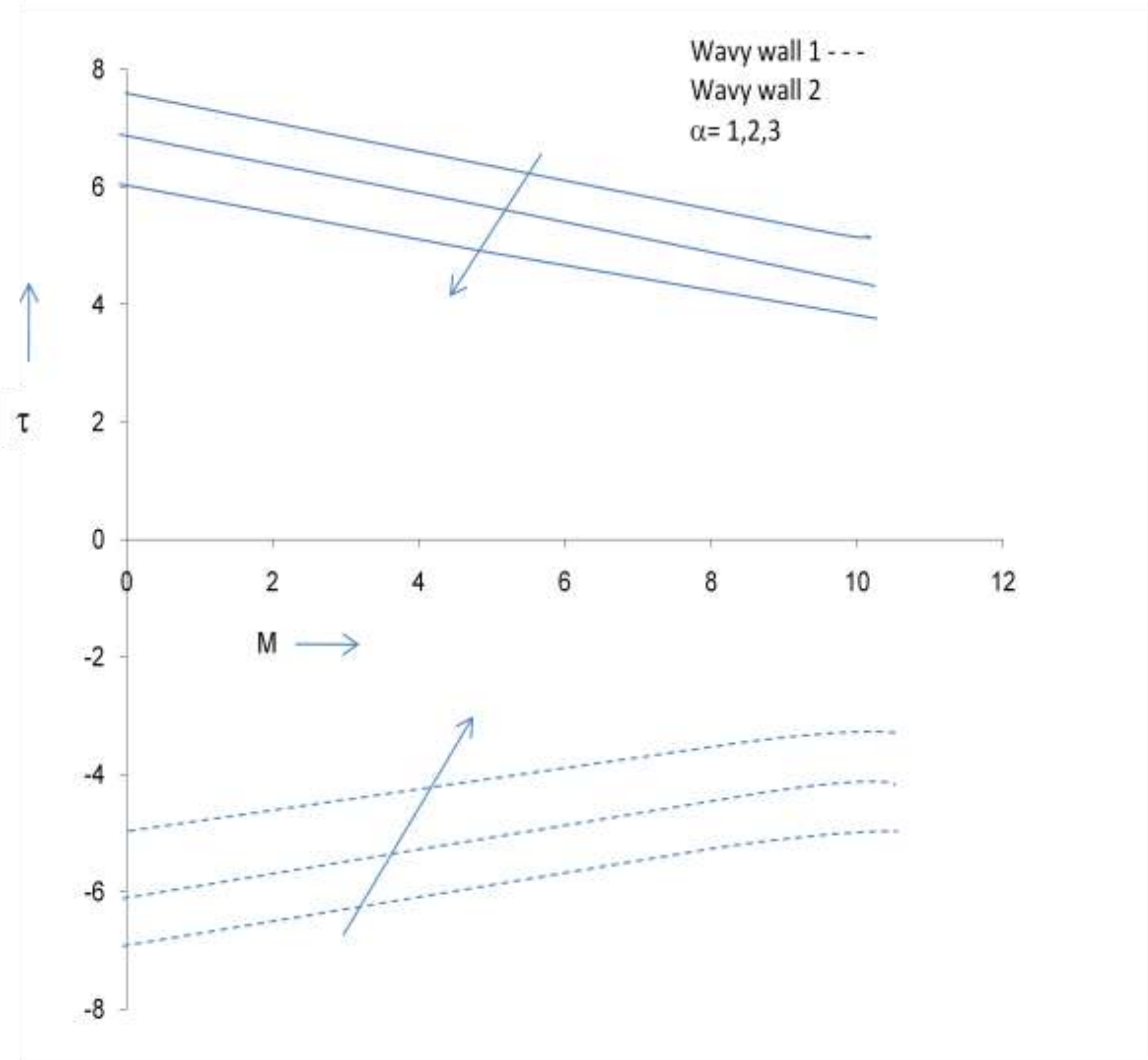


**Fig 6:** Skin friction □ □ against the Hartmann number M□ for Pr=0.71, Gr=2, Gm=3, m=.2, n=.4

International Journal of Advances in Engineering and Management (IJAEM)
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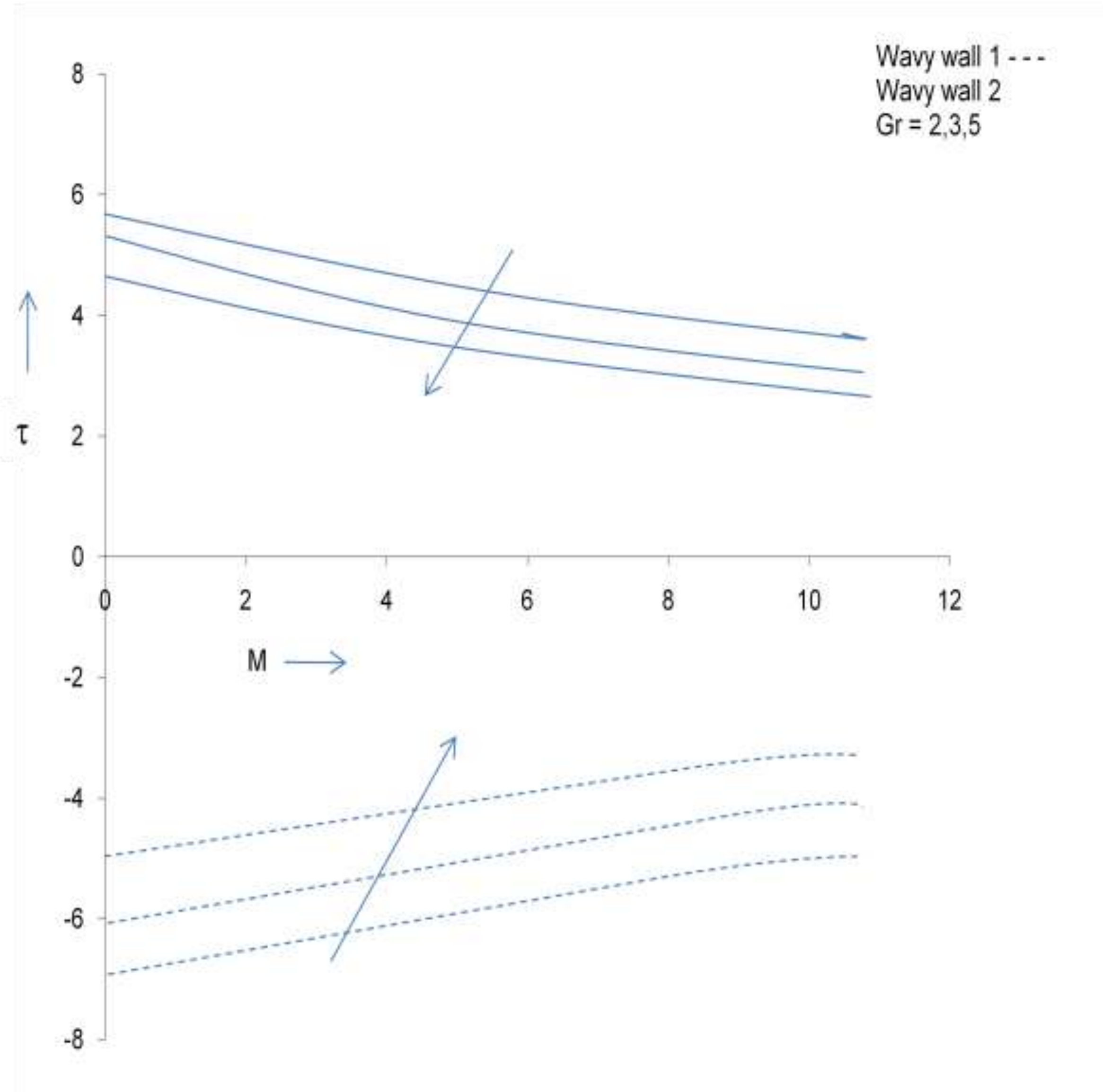


**Fig 7:** Skin friction  against the Hartmann number M for Pr=0.71, Gr=2, Gm=3, m=.2, n=.4, =1

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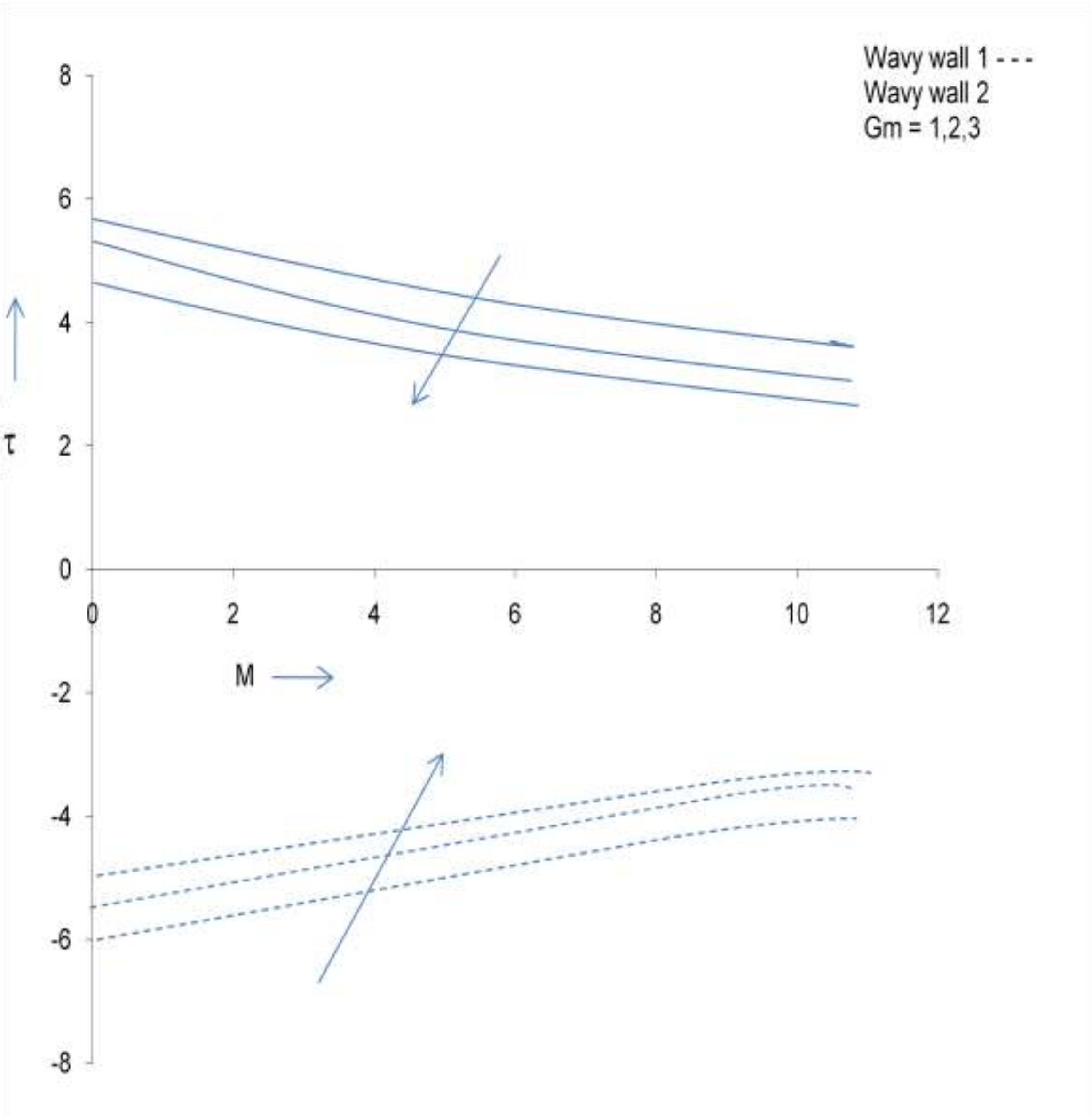


**Fig 8 :** Skin friction  against the Hartmann number M for Pr=0.71, Gr=2, m=.2, n=.4,  =1

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Volume 2, Issue 4, pp: 771-796 www.ijaem.net ISSN: 2395-5252

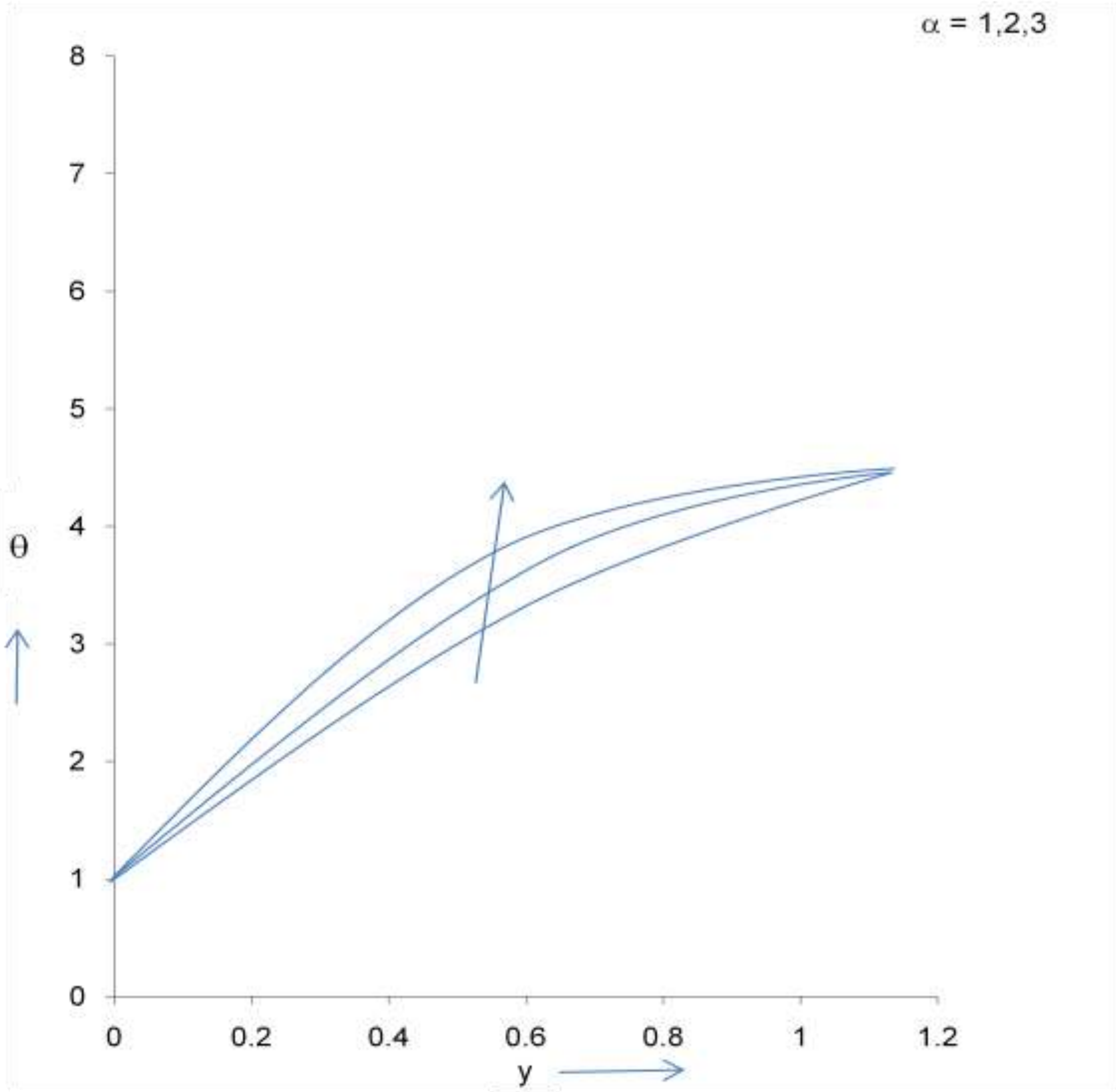


**Fig 9 :** Temperature profile for different values of□ □ □ for M=2, Gr=2, Gm=3, m=.2, n=.4, Pr=0.71

International Journal of Advances in Engineering and Management (IJAEM)
Volume 2, Issue 4, pp: 771-796 www.ijaem.net ISSN: 2395-5252

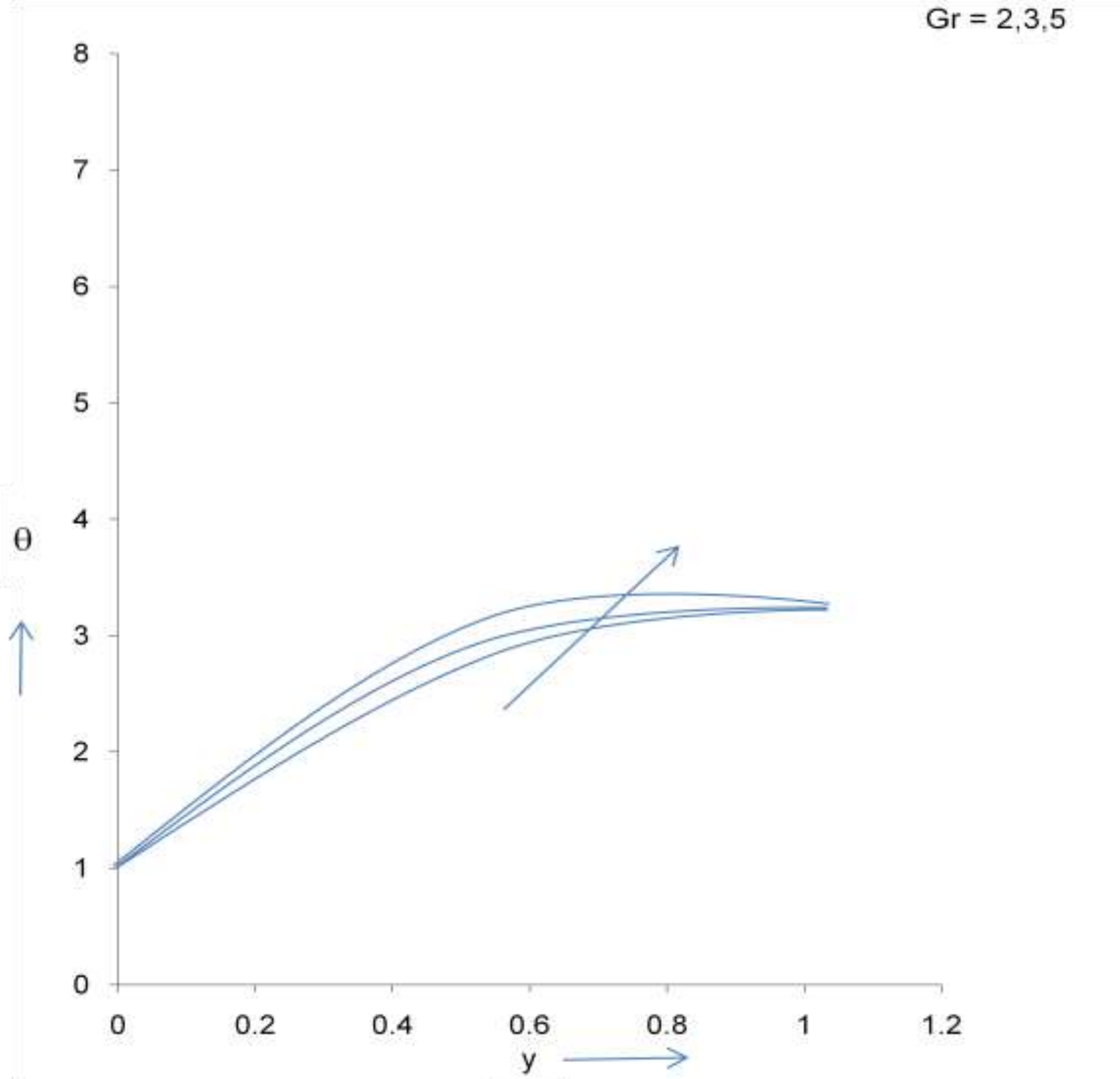


**Fig 10:** Temperature profile for different values of Gr for M=2, Gm=3, m=.2, n=.4, Pr=0.71, □ =1

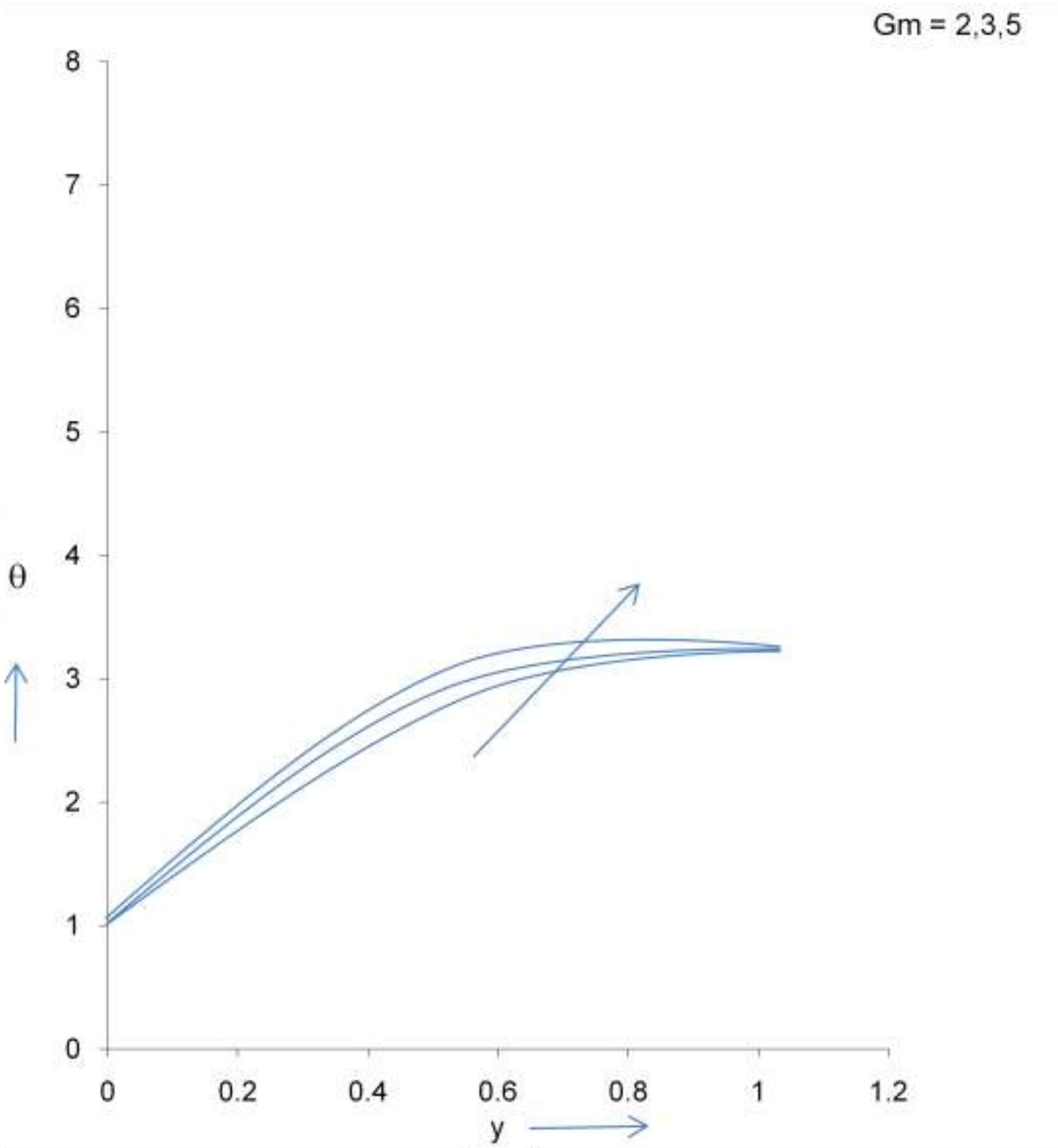


**Fig 11** : Temperature profile for different values of Gm for M=2, Gr=2, m=.2, n=.4, Pr=0.71, □ =1

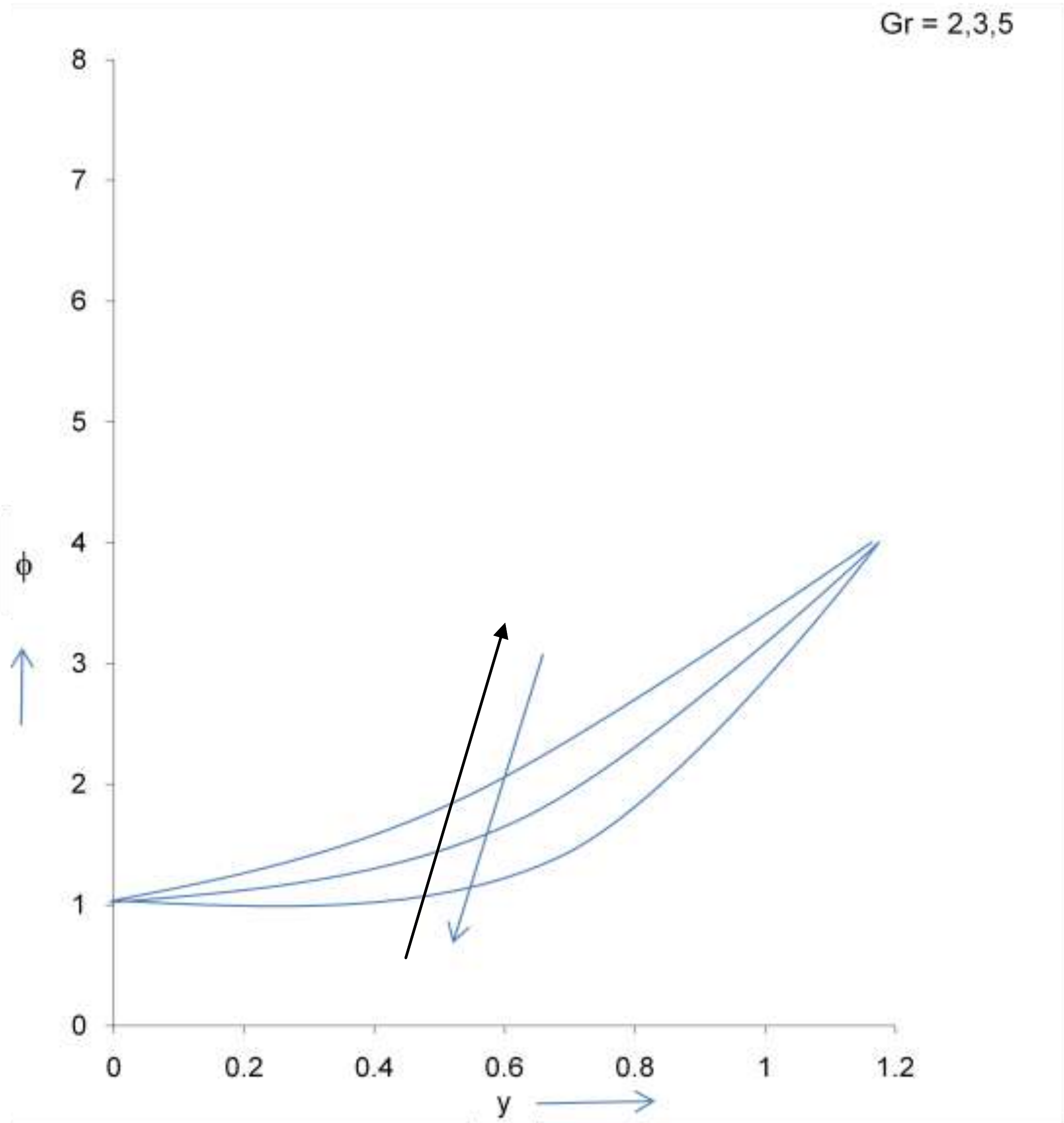


**Fig 12 :** Concentration profile for different values of Gr for M=2, Gm=3, m=.2, n=.4, Pr=0.71, □ =1

International Journal of Advances in Engineering and Management (IJAEM)
Volume 2, Issue 4, pp: 771-796 www.ijaem.net ISSN: 2395-5252

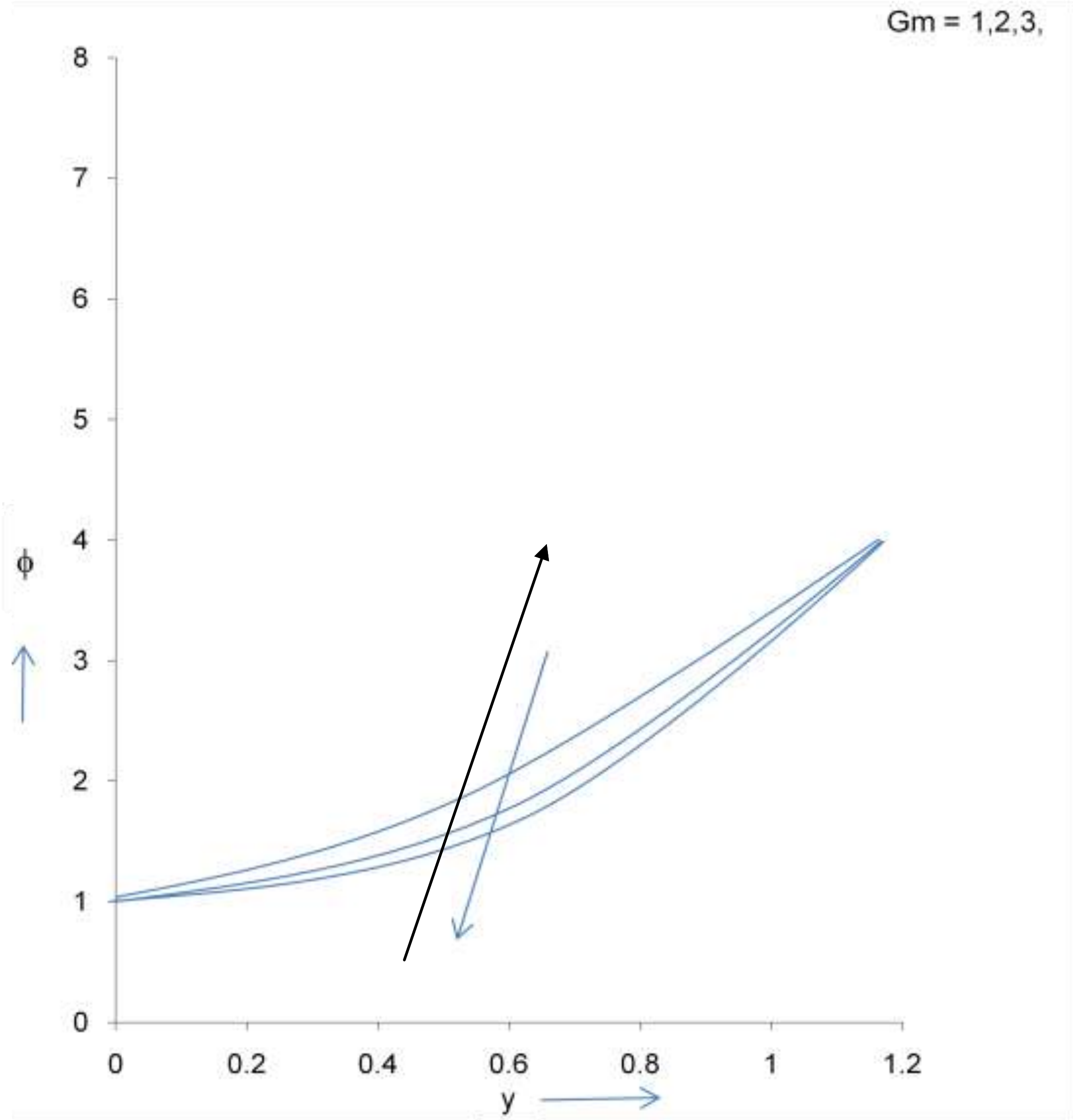


**Fig 13:** Concentration profile for different values of Gm for M=2, Gr=2, m=.2, n=.4, Pr=0.71, □ =1

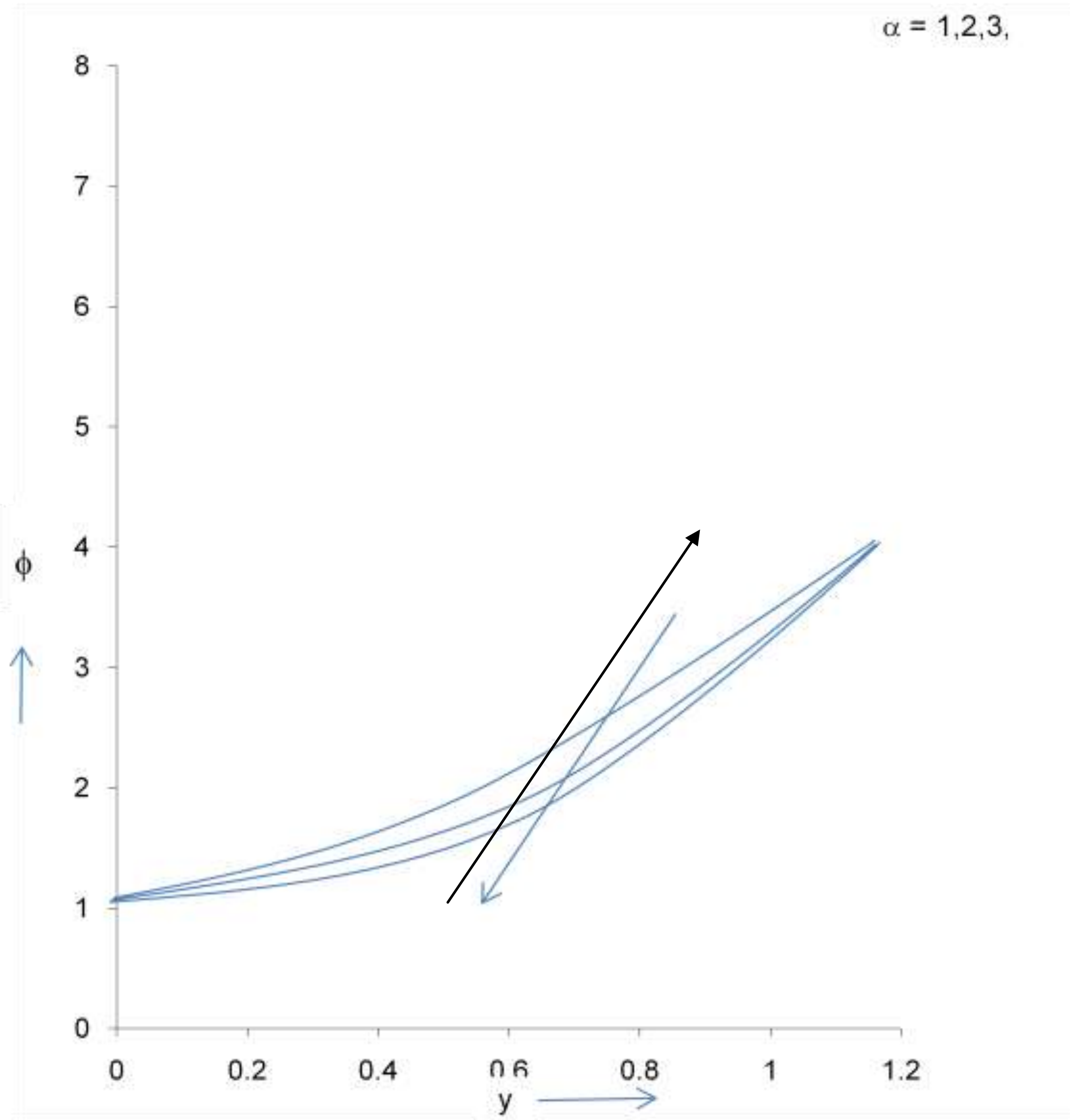


**Fig 14:** Concentration profile for different values of  for M=2, Gr=2, Gm=3,m=.2, n=.4, Pr=0.71

International Journal of Advances in Engineering and Management (IJAEM)
Volume 2, Issue 4, pp: 771-796 www.ijaem.net ISSN: 2395-5252

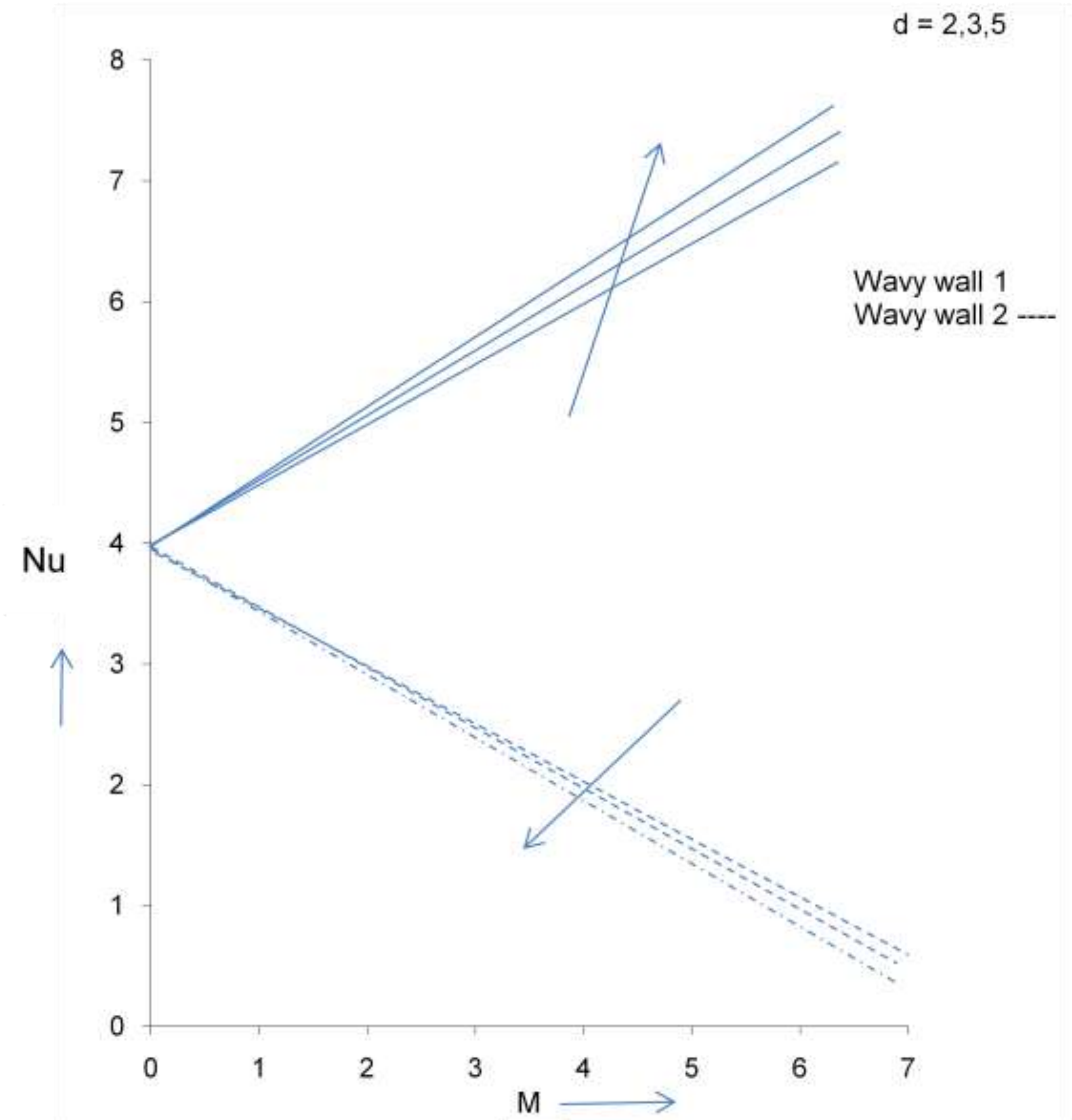


**Fig 15:** Nusselt number Nu against M for m=5, h=.5, Pr=0.71

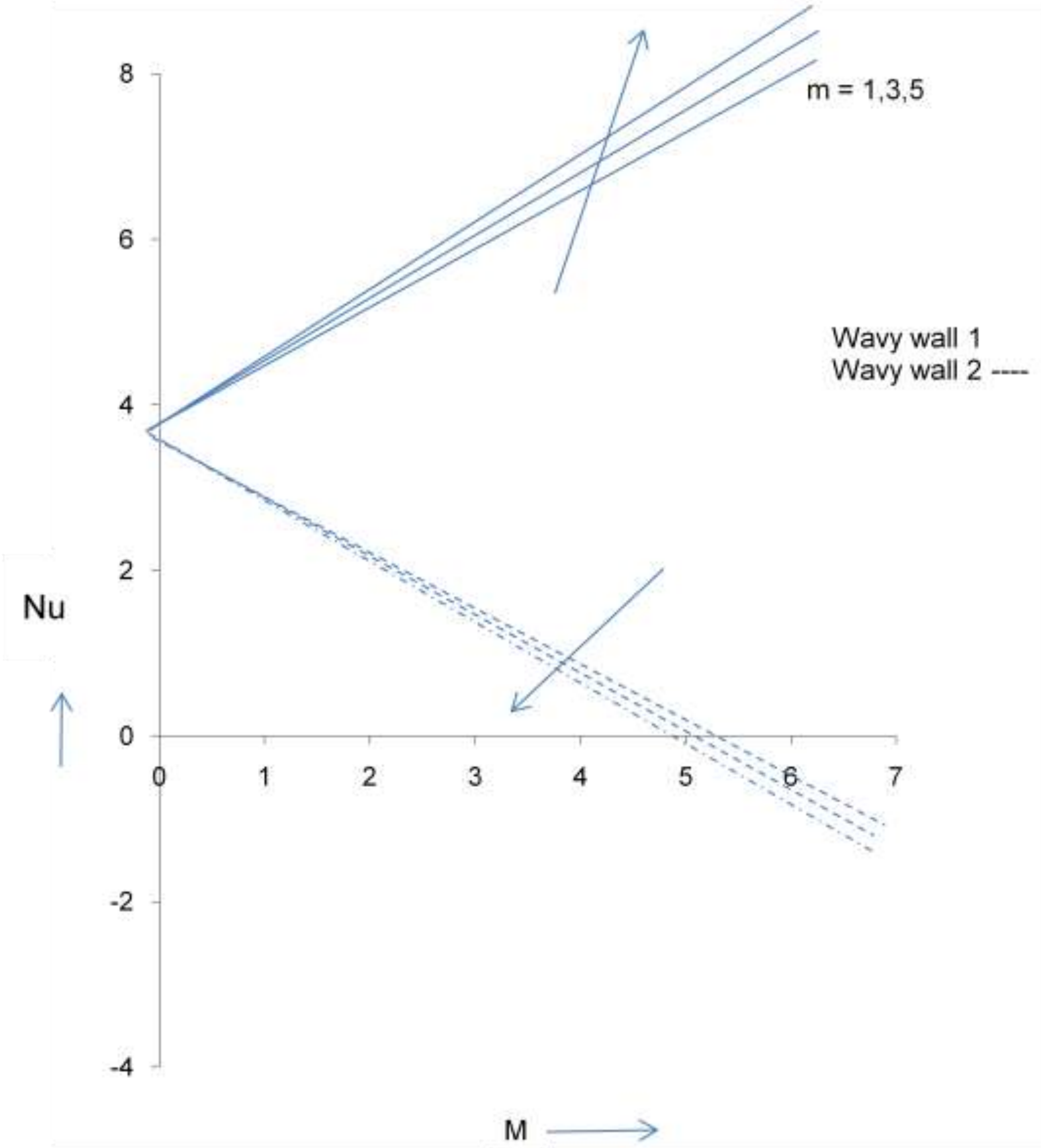


**Fig 16**: Nusselt number Nu against M for d=2, h=0.5, Pr=0.71

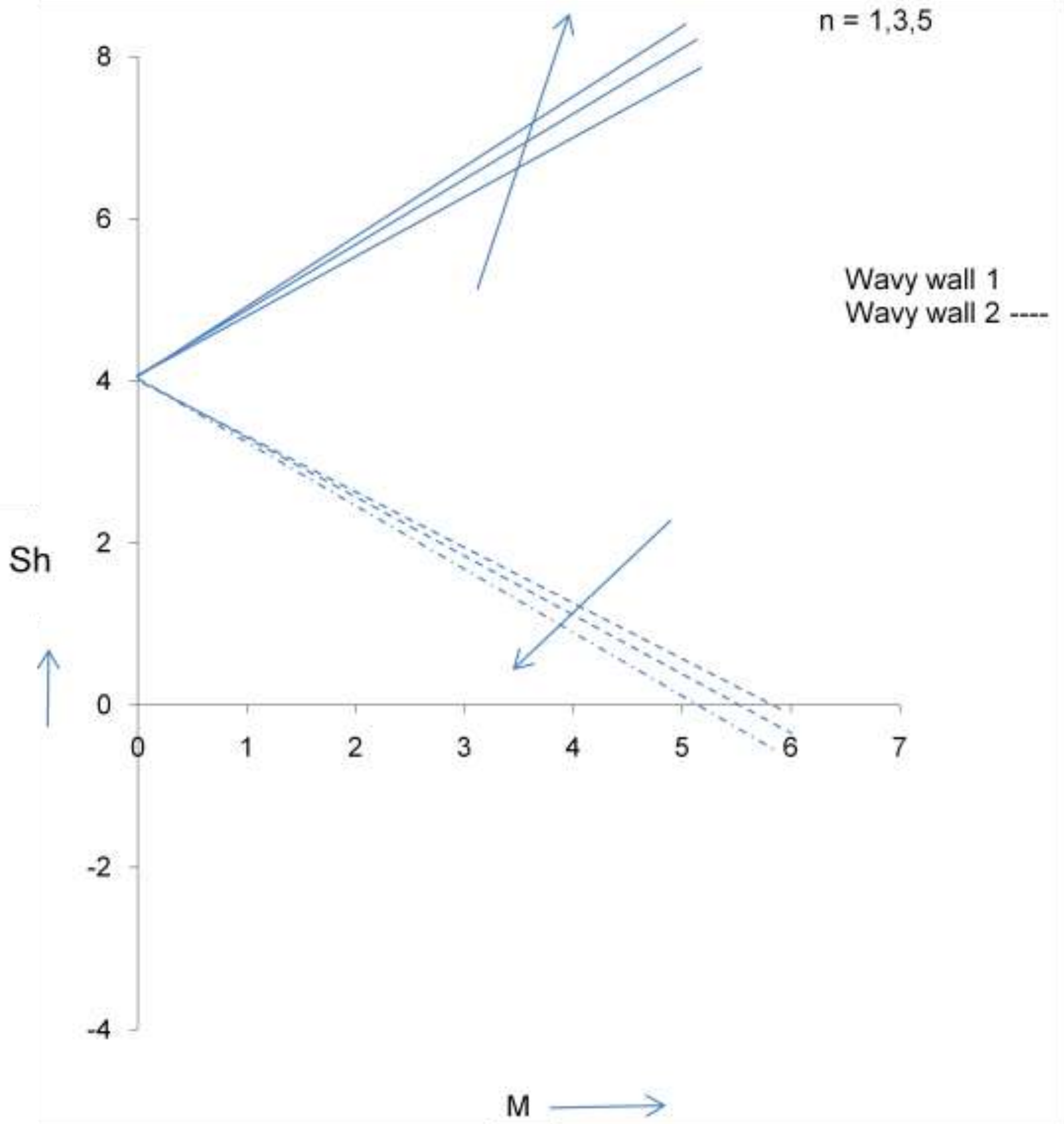


**Fig 17**: Sherwood number Sh against M for d=2, h=0.5, Pr=0.71

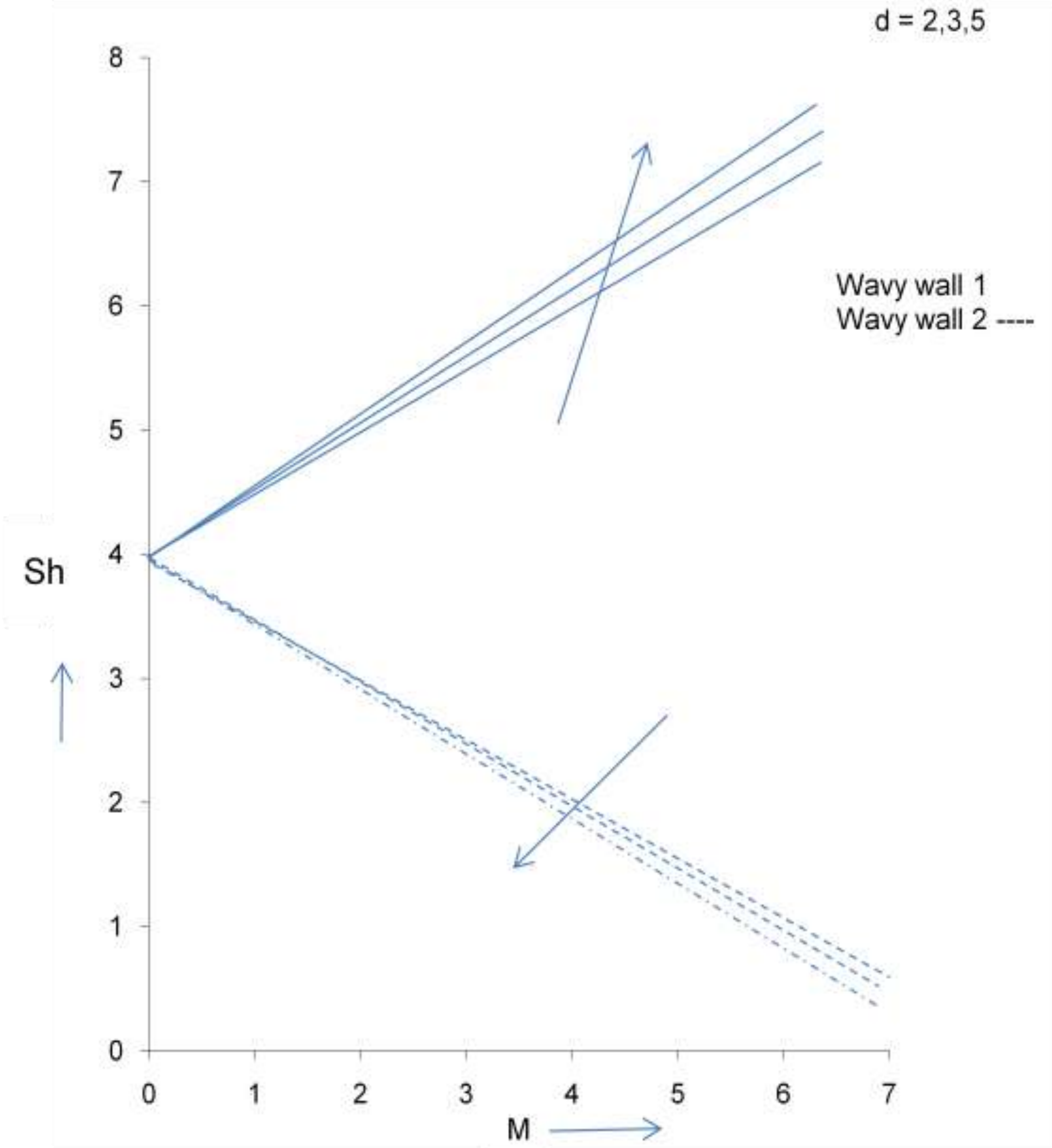


**Fig 18:** Sherwood number Sh against M for n=5, h=.5, Pr=0.71

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**Appendix:**

$$M_1=\left[Gr(m-1)+Gm(n-1)\right](\lambda_1-1),\ S_{10}=\frac{iGrM}{\lambda_1^4}+\frac{iGmM}{\lambda_1^4}+(Gr+Gm)Z_6,$$

$$S_{11}=\frac{GrM(m-1)}{\lambda_1^4}+iGr\Pr(1-m)Z_7+\frac{GmM(n-1)}{\lambda_1^4}+iGmPm(1-n)Z_7,$$

$$S_{12}=\frac{i\Pr(1-m)GrZ_8}{2}+\frac{i\,\mathrm{P}\,m(1-n)GmZ_8}{2},\ S_{13}=\frac{i\Pr(1-m)GrZ_9}{3}+\frac{ipPm(1-n)GmZ_9}{3},$$

$$S_{14}=iGrZ_3+iGmZ_3-iV_1\lambda_1^2Z_1+i\Pr(1-m)GrZ_{10}+iPm(1-n)GmZ_{10}-\frac{i\Pr(1-m)GrZ_{11}}{\lambda_1^2}-\frac{iPm(1-n)GmZ_{11}}{\lambda_1^2}+\frac{iV_1M}{\lambda_1^2},$$

$$S_{15}=iGrZ_4+iGmZ_4-iGm\lambda_1^2Z_1^2-iGr\lambda_1^2Z_1-\frac{iPr(1-m)GrZ_{12}}{\lambda_1^2}-\frac{iPm(1-n)GmZ_{12}}{\lambda_1^2}-\frac{iV_2M}{\lambda_1^2}-\frac{i\Pr Z_{13}}{\lambda_1^2}\left[Gr(1-m)+Gm(1-n)\right],$$

$$S_{16}=i(1-m)GrZ_3+i(1-n)GmZ_3+\frac{i\Pr Z_{11}}{\lambda_1^2}\left[Gr(1-m)+Gm(1-n)\right]-iGr\lambda_1^2Z_2-iGm\lambda_1^2Z_2,$$

$$S_{17}=i(1-m)GrZ_4+i(1-n)GmZ_4-iV_2\lambda_1^2Z_2-\frac{i\Pr(1-m)GrZ_{13}}{\lambda_1^2}-\frac{iPm(1-n)GmZ_{13}}{\lambda_1^2},$$

$$S_{18}=\frac{iV_1M}{2},S_{19}=\frac{-iGrM}{2}-\frac{iGmM}{2},Z_1=-Z_3-Z_4,Z_2=\lambda_1(Z_4-Z_3)+\lambda_2,$$

$$\lambda_2=\alpha(c_2-c_1)+\frac{Gr(m-1)}{\alpha^2}+\frac{Gm(n-1)}{\alpha^2},\lambda_3=\alpha_1\left[c_2e^{\alpha}\alpha-c_1e^{-\alpha}\alpha+\frac{Gr(m-1)}{\alpha^2}+\frac{Gm(n-1)}{\alpha^2}\right],$$

$$Z_3=\frac{\lambda_2}{S_1}-\frac{S_2}{S_2S_3+S_3S_1}\left[\frac{V_1+V_2}{2\lambda_1^2}+\frac{\lambda_2S_4}{S_1}\right],Z_4=\frac{S_1}{S_2S_3+S_3S_1}\left[\frac{V_1+V_2}{2\lambda_1^2}+\frac{\lambda_2S_4}{S_1}\right],$$

$$Z_{16}=Z_1+\frac{Gr}{\lambda_1^2}+\frac{Gm}{\lambda_1^2},Z_{17}=\frac{Gr}{\lambda_1^2}(m+\alpha_1+Z_2-2)+\frac{Gm}{\lambda_1^2}(n+\alpha_1+Z_2-2),$$

$$Z_{18}=\frac{M}{2\lambda_1^2}+\frac{Gr(m-1)}{\lambda_1^2}+\frac{Gm(n-1)}{\lambda_1^2},Z_{19}=V_1+Z_3,Z_{20}=Z_4+V_2,Z_{21}=(\alpha_1-1)V_1,Z_{22}=(\alpha_1-1)V_2,$$

$$Z_7=Z_1+\frac{Gr}{\lambda_1^2}+\frac{Gm}{\lambda_1^2},Z_8=\frac{Gr}{\lambda_1^2}(m+\alpha_1+Z_2-2)+\frac{Gm}{\lambda_1^2}(n+\alpha_1+Z_2-2),$$

$$Z_9=\frac{M}{2\lambda_1^2}+\frac{Gr(m-1)}{\lambda_1^2}+\frac{Gm(n-1)}{\lambda_1^2},Z_{10}=V_1+Z_3,Z_{11}=(\alpha_1-1)V_1,Z_{12}=Z_4+V_2,Z_{13}=(\alpha_1-1)V_2,$$

$$V_3=\frac{e^{\lambda_1}(1-\lambda_1)(1-e^{-\lambda_1})}{1-e^{\lambda_1}}+e^{-\lambda_1}(1+\lambda_1)\frac{D_{16}}{D_{15}}+D_{14}-D_{12},$$

$$V_4=\left[\frac{D_{16}}{D_{15}}-\frac{D_{16}}{D_{15}}y\frac{(1-e^{-\lambda_1})}{(1-e^{\lambda_1})}+\frac{D_{14}-D_{13}}{\lambda_1(1-e^{\lambda_1})}\right]\lambda_1-D_{13},V_6=\frac{S_{11}}{6\lambda_1^2}+\frac{S_{13}}{\lambda_1^2},V_7=\frac{S_{12}}{12\lambda_1^2},$$

$$V_8=\frac{S_{13}}{20\lambda_1^2},V_9=\frac{D_{16}}{D_{15}}\frac{(1-e^{-\lambda_1})}{(1-e^{\lambda_1})}+\frac{D_{14}-D_{13}}{\lambda_1(1-e^{-\lambda_1})},V_{10}=V_9+\frac{2S_{16}}{\lambda_1^5}-\frac{6S_{18}}{\lambda_1^6},V_{11}=-V_{12}-\frac{Gr}{\lambda_1^2}-\frac{Gm}{\lambda_1^2},$$

$$V_{12}=\frac{Gr(m-e^{\lambda_1})+Gm(n-e^{\lambda_1})}{2\lambda_1^2\sinh\lambda_1},V_{13}=\frac{S_{14}}{2\lambda_1^3}-\frac{5S_{18}}{4\lambda_1^4}+\frac{17S_{18}}{4\lambda_1^5},V_{14}=\frac{S_{15}}{2\lambda_1^3}-\frac{3S_{17}}{3\lambda_1^4}-\frac{17S_{19}}{4\lambda_1^5},$$

$$D_{11}=V_{10}+V_{12},D_{12}=-V_{19}-V_6-V_7-V_8+V_{10}e^{\lambda_1}+V_{12}e^{-\lambda_1}+V_{13}e^{\lambda_1}-V_{14}e^{-\lambda_1}+V_{15}e^{\lambda_1}-V_{16}e^{-\lambda_1},$$

$$D_{13}=\lambda_1V_{10}-\lambda_1V_{12}+V_{13}-V_{14},$$

$$D_{14}=-2V_{19}-3V_6-4V_7-5V_{10}+\lambda_1V_{10}e^{\lambda_1}+(1+\lambda_1)V_{13}e^{\lambda_1}-(1-\lambda_1)V_{14}e^{-\lambda_1}+(2+\lambda_1)V_{15}e^{\lambda_1}$$
$$-(2-\lambda_1)V_{16}e^{-\lambda_1}+(3+\lambda_1)V_{19}e^{\lambda_1}-(3-\lambda_1)V_{18}e^{-\lambda_1},$$

$$D_{15}=1+\frac{(1-e^{\lambda_1})}{(1-e^{-\lambda_1})}-\frac{e^{\lambda_1}(1-\lambda)(1-e^{\lambda_1})}{1-e^{\lambda_1}},D_{16}=D_{12}-D_{14}-D_{11}-\frac{D_{14}-D_{13}}{\lambda_1(1-e^{\lambda_1})}+\frac{e^{\lambda_1}(1-\lambda_1)(D_{14}-D_{13})}{\lambda_1(1-e^{\lambda_1})}$$

.

$V_1$=Gr(m-1) $(\lambda_1-1)$, $V_2$=Gm(n-1) $(\lambda_1-1)$,